

# Casimir Effect : Optomechanics in Quantum Vacuum

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The Casimir effect results from the optomechanical coupling between field fluctuations and mirrors in quantum vacuum. This contribution to the 20th International Conference on Laser Spectroscopy (ICOLS 2011) discusses the current status in the comparison between theory and experiments.

## I. INTRODUCTION

The Casimir effect [1] is a jewel with many facets. First, it is a macroscopic effect of the irreducible field fluctuations which fill quantum vacuum. As a crucial prediction of quantum theory, it has thus been the focus of a number of works (see reviews in [2–8]).

Then, it has fascinating interfaces with some of the most important open questions in fundamental physics. It is connected with the puzzles of gravitational physics through the problem of vacuum energy [9, 10] as well as with the principle of relativity of motion through the dynamical Casimir-like effects [11]. Effects beyond the Proximity Force Approximation also make apparent the rich interplay of vacuum energy with geometry [12–14] (more discussions below).

Casimir physics also plays an important role in the tests of gravity at sub-millimeter ranges [15, 16]. Strong constraints have been obtained in short range Cavendish-like experiments [17]. Should an hypothetical new force have a Yukawa-like form, its strength could not be larger than that of gravity if the range is larger than  $56\mu\text{m}$ . For scales of the order of the micrometer, gravity tests are performed by comparing the results of Casimir force measurements with theory [18–20]. Other constraints can be obtained with atomic or nuclear force measurements (for a recent overview of short-range tests, see [21]).

Finally, the Casimir force and closely related Van der Waals force are dominant at micron or sub-micron distances. This entails that they have strong connections with various active domains and interfaces of physics, such as atomic and molecular physics, condensed matter and surface physics, chemical and biological physics, micro- and nano-technology [22]. In the following, we will stress that Casimir physics reveals optomechanical couplings of macroscopic mirrors with quantum vacuum fields.

## II. THE PUZZLE OF VACUUM ENERGY

The classical idealization of space as being absolutely empty was already affected by the advent of statistical mechanics, when it was realized that space is in fact filled with black body radiation. The first quantum law was designed by Planck precisely to explain the properties of this black body radiation [23]. In modern terms, it gave the mean energy per electromagnetic mode as the prod-

uct  $\bar{n}\hbar\omega$  of the photon energy  $\hbar\omega$  by the mean number of photons per mode  $\bar{n} = \left(\exp \frac{\hbar\omega}{k_B T} - 1\right)^{-1}$ .

Like Einstein, Planck was aware of the unsatisfactory character of his derivation. Among other physicists, they attempted for years to give more satisfactory treatments by studying in more detail the interaction between matter and radiation. These attempts led to the discovery by Einstein of the quantum absorption-emission laws and of the Bose statistics (see [24–26]). In 1911, Planck [27] wrote a new expression for the mean energy per mode  $(\bar{n} + \frac{1}{2})\hbar\omega$  which contained a zero-point energy  $\frac{1}{2}\hbar\omega$  besides the black body energy. In contrast to the latter, the zero-point fluctuations were still present at zero temperature. The arguments thus used by Planck cannot be considered as consistent today. The first known argument still acceptable today was proposed by Einstein and Stern [28] in 1913 : the second Planck law (but not the first one) reproduces the classical limit  $(\bar{n} + \frac{1}{2})\hbar\omega = k_B T + O(\frac{1}{T})$  at high temperatures  $T \rightarrow \infty$ . Amazingly, this argument fixes the magnitude of zero-point fluctuations, essentially visible at low temperatures, by requiring their disappearance at high temperatures to be as perfect as possible !

Some physicists took zero-point fluctuations seriously, long before the advent of the fully developed quantum theory. Debye insisted on observable consequences of zero-point atomic motions, in particular through their effects on the intensities of diffraction peaks [29]. Mulliken produced experimental evidence of the effects of zero-point motions by studying isotopic shifts in vibrational spectra of molecules [30]. Nernst was the first physicist to notice, in 1916, that zero-point fluctuations of the electromagnetic field constituted a challenge for gravitation theory [31, 32]. When the energy density is calculated by summing up the energies over all field modes, a finite value is obtained for the first Planck law (this is the solution of the ‘ultraviolet catastrophe’) but an infinite value is produced from the second law. When a high frequency cutoff  $\omega_{\text{max}}$  is introduced, the calculated energy density  $\frac{(\hbar\omega_{\text{max}})^4}{8\pi^2(\hbar c)^3}$  is finite but still much larger than the mean energy observed in the world around us through gravitational phenomena [33]. The ratio of calculated to observed energy density has a huge value, up to  $10^{120}$  in the most extreme estimations [34].

This major problem, known since 1916 and still unsolved today, has led famous physicists to deny the reality of vacuum fluctuations. In particular, Pauli stated in his

textbook on Wave Mechanics [35] : *At this point it should be noted that it is more consistent here, in contrast to the material oscillator, not to introduce a zero-point energy of  $\frac{1}{2}\hbar\omega$  per degree of freedom. For, on the one hand, the latter would give rise to an infinitely large energy per unit volume due to the infinite number of degrees of freedom, on the other hand, it would be principally unobservable since nor can it be emitted, absorbed or scattered and hence, cannot be contained within walls and, as is evident from experience, neither does it produce any gravitational field.* A part of these statements is simply unescapable : it is just a matter of evidence that the mean value of vacuum energy does not contribute to gravitation as an ordinary energy. But it is certainly no longer possible to uphold today that vacuum fluctuations have no observable effects. Certainly, vacuum fluctuations are *scattered* by matter, as shown by their numerous effects in atomic [36] and subatomic [37] physics. And the Casimir effect is nothing but the evidence of vacuum fluctuations making their existence manifest when being *contained within walls*.

### III. THE CASIMIR FORCE

Casimir calculated the force between a pair of perfectly smooth, flat and parallel plates in the limit of zero temperature and perfect reflection. In this idealized case, the expressions for the force  $F_{\text{Cas}}$  and energy  $E_{\text{Cas}}$  reveal a universal effect resulting from the confinement of vacuum fluctuations

$$F_{\text{Cas}} = -\frac{dE_{\text{Cas}}}{dL} \quad , \quad E_{\text{Cas}} = -\frac{\hbar c \pi^2 A}{720 L^3} \quad (1)$$

with  $L$  the distance,  $A$  the area,  $c$  the speed of light and  $\hbar$  the Planck constant. This universality is explained by the saturation of the optical response of mirrors reflecting 100% of incoming fields. In particular the expressions  $F_{\text{Cas}}$  and  $E_{\text{Cas}}$  do not depend on the atomic structure constants.

This idealization is no longer tenable for the real mirrors used in the experiments. It is thus necessary to take into account the optical properties of these mirrors [38, 39]. The most precise experiments have been performed with metallic mirrors which are good reflectors only at frequencies smaller than their plasma frequency  $\omega_P$ . Their optical response is described by a reduced dielectric function written at imaginary frequencies  $\omega = i\xi$  as

$$\varepsilon[i\xi] = \hat{\varepsilon}[i\xi] + \frac{\sigma[i\xi]}{\xi} \quad , \quad \sigma[i\xi] = \frac{\omega_P^2}{\xi + \gamma} \quad (2)$$

The function  $\hat{\varepsilon}[i\xi]$  represents the contribution of inter-band transitions and is regular at the limit  $\xi \rightarrow 0$ . Meanwhile  $\sigma[i\xi]$  is the reduced conductivity ( $\sigma$  is measured as a frequency and the SI conductivity is  $\epsilon_0\sigma$ ) which describes the contribution of the conduction electrons.

A simplified description corresponds to the lossless limit  $\gamma \rightarrow 0$  often called the plasma model. As  $\gamma$  is much smaller than  $\omega_P$  for a metal such as Gold, this simple model captures the main effect of imperfect reflection. However it cannot be considered as an accurate description since a much better fit of tabulated optical data is obtained with a non null value of  $\gamma$ . When taking into account the imperfect reflection of the metallic mirrors, one finds that the Casimir force is reduced with respect to the ideal Casimir expression at all distances for a null temperature. This reduction is conveniently represented as a factor  $\eta_F = F/F_{\text{Cas}}$  where  $F$  is the real force and  $F_{\text{Cas}}$  the ideal expression. For the plasma model, there is only one length scale, the plasma wavelength  $\lambda_P = 2\pi c/\omega_P$  (136nm for Gold). The ideal Casimir formula is recovered ( $\eta_F \rightarrow 1$ ) at large distances  $L \gg \lambda_P$ , as expected from the fact that metallic mirrors tend to be perfect reflectors at low frequencies  $\omega \ll \omega_P$ . At short distances in contrast, a significant reduction of the force is obtained ( $\eta_F \propto L/\lambda_P$ ), as a consequence of the fact that metallic mirrors are poor reflectors at high frequencies  $\omega \gg \omega_P$ . In other words, there is a change in the power law for the variation of the force with distance. This change can be understood as the result of the Coulomb interaction of surface plasmons living at the two matter-vacuum interfaces [40, 41].

Experiments are performed at room temperature so that the effect of thermal fluctuations has to be added to that of vacuum fields [42, 43]. Significant thermal corrections appear at distances  $L$  larger than a critical distance determined by the thermal wavelength  $\lambda_T$  (a few micrometers at room temperature). Boström and Sernelius were the first to remark that the small non zero value of  $\gamma$  had a significant effect on the force at non null temperatures [44]. In particular, there is a large difference at large distances between the expectations calculated for  $\gamma = 0$  and  $\gamma \neq 0$ , their ratio reaching a factor 2 when  $L \gg \lambda_T$ . It is also worth emphasizing that the contribution of thermal fluctuations to the force is opposite to that of vacuum fluctuations for intermediate ranges  $L \sim \lambda_T$ . This situation has led to a blossoming of contradictory papers (see references in [45–47]). As we will see below, the contradiction is also deeply connected to the comparison between theory and experiments.

Another important feature of the recent precise experiments is that they are performed in the geometry of a plane and a sphere. The estimation of the force in this geometry uses the so-called *Proximity Force Approximation* (PFA) [48] which amounts to integrating over the distribution of local inter-plate distances the pressure calculated in the geometry with two parallel planes. But Casimir forces are certainly not additive ! The PFA can only be valid when the radius  $R$  of the sphere is much larger than the separation  $L$  between the plane and the sphere. Even in this case its accuracy remains a question of importance for the comparison between theory and experiments.

We now discuss the status of comparisons between

Casimir experiments and theory. After years of improvement on both sides, we have to face discrepancies in these comparisons : there are differences between experimental results and theoretical predictions drawn from the expected models, as well as disagreements between some recent experiments.

On one hand, there have been experiments in Purdue and Riverside for approximately ten years, the results of which point to an unexpected conclusion (see [49–51]). The Purdue experiment uses dynamic measurements of the resonance frequency of a microresonator. The shift of the resonance gives the gradient of the Casimir force in the plane-sphere geometry, which is also (within PFA) the Casimir pressure between two planes. The typical radius of the sphere is  $R = 150\mu\text{m}$  and the range of distances  $L = 0.16 - 0.75\mu\text{m}$ . The results appear to fit predictions obtained from the lossless plasma model  $\gamma = 0$  rather than those corresponding to the expected dissipative Drude model  $\gamma \neq 0$  (see Fig.1 in [50]), in contradiction with the fact that Gold has a finite static conductivity  $\sigma_0 = \omega_P^2/\gamma$ . Note that these experiments are performed at distances where the thermal contribution as well as the effect of  $\gamma$  are not so large, so that the estimation of accuracy is a critical issue in these experiments.

On the other hand, there is now a new experiment in Yale [52], where a much larger sphere  $R = 156\text{mm}$  is used, allowing for measurements at larger distances  $L = 0.7 - 7\mu\text{m}$ . The thermal contribution is large there and the difference between the predictions at  $\gamma = 0$  and  $\gamma \neq 0$  is significant. Another problem appears which is the large contribution of the electrostatic patch effect [53–57]. After subtraction of this contribution of the patch effect, the results of the Yale experiment fit the expected Drude model. Of course, these new results have to be confirmed by further studies [58].

The conclusion of this discussion is that the Casimir effect, now measured in several experiments, is however not tested at the 1% level, as has been sometimes claimed. In particular, the patch effect remains a source of concern for Casimir experiments, as for other precision measurements (see examples in [59–67]). The patch distribution has not been measured in any of the experiments discussed above and progress could of course come with better characterization and control of surfaces. The related problem of surface roughness has also to be studied in more detail [68–70]. We note also that the aspect ratio  $L/R$  lies in the range  $[10^{-3}, 5 \times 10^{-3}]$  for the Purdue experiment,  $[5 \times 10^{-5}, 5 \times 10^{-6}]$  for the Yale experiment, so that the corrections to PFA could have quite different effects in the two cases.

#### IV. THE CASIMIR EFFECT IN THE SCATTERING APPROACH

The best tool available for addressing these questions is the scattering approach. This approach has been used

for years for evaluating the Casimir force between non perfectly reflecting mirrors. It is today the best solution for calculating the force in arbitrary geometries [71].

The basic idea is that mirrors are described by their scattering amplitudes. It can be simply illustrated with the model of scalar fields propagating along the two directions on a line (1-dimensional space; see references in [11]). Each mirror is described by a scattering matrix containing reflection and transmission amplitudes. Two mirrors form a Fabry-Perot cavity described by a scattering matrix  $S$  which can be deduced from the two elementary matrices. The Casimir force then results from the difference of radiation pressures exerted onto the inner and outer sides of the mirrors by the vacuum field fluctuations [72]. Equivalently, the Casimir free energy can be derived from the frequency shifts of all vacuum field modes due to the presence of the cavity.

The same discussion can be extended to the geometry of two plane and parallel mirrors aligned along the axis  $x$  and  $y$ , described by specular reflection and transmission amplitudes which depend on frequency  $\omega$ , the transverse vector  $\mathbf{k} \equiv (k_x, k_y)$  and the polarization  $p = \text{TE, TM}$ . A few points have to be treated with care when extending the derivation from 1-dimensional space to 3-dimensional space : evanescent waves contribute besides ordinary modes freely propagating outside and inside the cavity; dissipation has to be accounted for [73]. The properties of the evanescent waves are described through an analytical continuation of those of ordinary ones, using the well defined analytic behavior of the scattering amplitudes. At the end of this derivation, this analytic properties are also used to perform a Wick rotation from real to imaginary frequencies.

The sum of all phaseshifts leads to the expression of the Casimir free energy  $\mathcal{F}$

$$\mathcal{F} = \sum_{\mathbf{k}} \sum_p k_B T \sum_m' \ln d(i\xi_m, \mathbf{k}, p) \quad (3)$$

$$d = 1 - r e^{-2KL}, \quad \xi_m \equiv \frac{2\pi m k_B T}{\hbar}, \quad K \equiv \sqrt{\mathbf{k}^2 + \frac{\xi^2}{c^2}}$$

$\sum_{\mathbf{k}} \equiv A \int \frac{d^2\mathbf{k}}{4\pi^2}$  is the sum over transverse wavevectors with  $A$  the area of the plates,  $\sum_p$  the sum over polarizations and  $\sum_m'$  the Matsubara sum (sum over positive integers  $m$  with  $m = 0$  counted with a weight  $\frac{1}{2}$ );  $d$  is the denominator describing cavity resonances;  $r \equiv r_1 r_2$  is the product of the reflection amplitudes of the mirrors as seen by the intracavity field;  $\xi$  and  $K$  are the counterparts of frequency  $\omega$  and longitudinal wavevector  $k_z$  after the Wick rotation.

This expression reproduces the Casimir ideal formula in the limits of perfect reflection  $r \rightarrow 1$  and null temperature  $T \rightarrow 0$ . But it is valid and regular at thermal equilibrium at any temperature and for any optical model of mirrors obeying causality and high frequency transparency properties. It can thus be used for calculating the Casimir force between arbitrary mirrors, as soon as the reflection amplitudes are specified. These amplitudes

are commonly deduced from models of mirrors, the simplest of which is the well known Lifshitz model [74, 75] which corresponds to semi-infinite bulk mirrors characterized by a local dielectric response function  $\varepsilon(\omega)$  and reflection amplitudes deduced from the Fresnel law. In principle, the expression (3) can still be written in terms of reflection amplitudes even when the optical response of the mirrors can no longer be described by a local dielectric response function.

## V. THE NON SPECULAR SCATTERING APPROACH

The scattering formalism can be generalized one step further to calculate the Casimir force between stationary objects with arbitrary geometries. Now the scattering matrix  $\mathcal{S}$  is a larger matrix accounting for non-specular reflection and mixing different wavevectors and polarizations while preserving frequency. Of course, the non-specular scattering formula is the generic one while the specular limit can only be an idealization.

The Casimir free energy can be written as a generalization of equation (3)

$$\mathcal{F} = k_B T \sum_m' \text{Tr} \ln \mathcal{D}(i\xi_m) \quad (4)$$

$$\mathcal{D} = 1 - \mathcal{R}_1 \exp^{-\kappa L} \mathcal{R}_2 \exp^{-\kappa L}$$

The symbol  $\text{Tr}$  refers to a trace over the modes at a given frequency. The matrix  $\mathcal{D}$  is the denominator containing all the resonance properties of the cavity formed by the two objects 1 and 2 here written for imaginary frequencies. It is expressed in terms of the matrices  $\mathcal{R}_1$  and  $\mathcal{R}_2$  which represent reflection on the two objects 1 and 2 and of propagation factors  $\exp^{-\kappa L}$ . Note that the matrices  $\mathcal{D}$ ,  $\mathcal{R}_1$  and  $\mathcal{R}_2$ , which were diagonal on the basis of plane waves when they described specular scattering, are no longer diagonal in the general case of non specular scattering. The propagation factors remain diagonal in this basis with their diagonal values written as in (3). Clearly the expression (4) does not depend on the choice of a specific basis. But it may be written in specific basis fitting the geometry under study.

The multiple scattering formalism has been used in the past years by different groups using different notations (see as examples [76–78]). Numerous applications have been considered and some of them are discussed in the next section.

## VI. APPLICATIONS TO DIFFERENT GEOMETRIES

Various geometries can be studied beyond the PFA by using the general expression (4). For example, one can study plane or spherical plates, flat, rough or nanostructured surfaces, atoms, molecules or nanoparticles, as well as different combinations of these possibilities.

The first applications were devoted to the effect of surface roughness on the normal Casimir force [79–81]. The case of corrugated plates is also interesting, in particular because it gives rise to lateral forces when the corrugations are shifted with respect to each other [82–84] and to torques when they are misaligned [85]. In the geometry of a sphere above a grating, the normal Casimir force is affected in a manner which can be understood quantitatively by using the non specular scattering approach [86–89].

Applications have also been developed for the study of atoms in the vicinity of corrugated plates [90, 91]. Examples of such applications involve the use of Bose-Einstein condensates (BEC) as probes of vacuum affected by the proximity of a grooved surface [92], localization of matter waves in the disordered vacuum above a rough plate [93], and the driving of quantized vortices in a BEC by a contactless transfer of angular momentum through the mediation of vacuum fluctuations [94]. All these applications involve Casimir forces or torques beyond the regime of validity of the PFA.

Efforts have also been devoted to the use of multiple scattering method to obtain explicit evaluations of the Casimir force in the plane-sphere geometry. Such calculations have first been performed for perfectly reflecting mirrors [95]. They have then been done for the more realistic case of metallic mirrors described by a plasma model dielectric function [96]. More recently, it has become possible to make calculations which treat simultaneously plane-sphere geometry and non zero temperature, with dissipation taken into account [97]. In these calculations, the reflection matrices are written in terms of Fresnel amplitudes for plane waves on the plane mirror and of Mie amplitudes for spherical waves on the spherical mirror. The scattering formula is then obtained by writing transformation formulas from the plane waves basis to the spherical waves basis and conversely. The energy takes the form of an exact multipolar formula labeled by a multipolar index  $\ell$ . When doing the numerics, the expansion is truncated at some maximum value  $\ell_{\max}$ , which degrades the accuracy of the resulting estimation for very large spheres  $x \equiv L/R < x_{\min}$  with  $x_{\min}$  proportional to  $\ell_{\max}^{-1}$ .

The results of these calculations may be compared to the only experiment devoted to the study of PFA in the plane-sphere geometry [98]. In this experiment, the force gradient  $G$  was measured for various radii of the sphere and the results were used to constrain the value of the slope at origin  $\beta_G$  of the function

$$\rho_G(x) = \frac{G}{G_{\text{PFA}}} = 1 + \beta_G x + O(x^2) \quad , \quad x \equiv \frac{L}{R} \quad (5)$$

where  $x$  is the already discussed aspect ratio which characterizes the plane-sphere geometry. The constraint obtained in this experiment is read  $|\beta_G| < 0.4$ . Its comparison with the theoretical value obtained for the slope  $\beta_G$  by interpolating at low values of  $x$  the numerically evaluated  $\rho_G$  reveals a striking difference between the cases of

perfect and plasma mirrors. The slope  $\beta_G^{\text{perf}} \sim -0.48$  obtained for perfect mirrors is indeed not compatible with the experimental bound. In contrast, the slope  $\beta_G^{\text{Gold}} \sim -0.21$  obtained for more precisely described gold mirrors is significantly smaller and, as a result, compatible with the experimental bound.

The effect of temperature is also correlated with that of plane-sphere geometry. The first calculations accounting simultaneously for plane-sphere geometry, temperature and dissipation [97] show several striking features. The factor 2 between two planes at long distances calculated with Drude and plasma models is reduced to a factor below 3/2 in the plane-sphere geometry. Then, PFA underestimates the Casimir force within the Drude model at short distances, but overestimates it at all distances for the perfect reflector and plasma model. If the latter feature were conserved for the aspect ratios met in the experiments, the actual values of the Casimir force calculated within plasma and Drude model could be closer than what PFA suggests. This would affect the comparison of Casimir measurements with theory, which is still based on calculations using PFA.

We finally refer to the study of the Casimir interaction

between a dielectric nanosphere and a metallic plane [99]. The known Casimir-Polder formula is recovered at the limit of small nanospheres, which may be thought of as large atoms. Meanwhile an expression that takes into account the finite size of the sphere is found, which behaves better at small distances than the Casimir-Polder formula. This opens the way to new studies devoted to the optomechanics of nanoobjects in the vacuum modified by the proximity of a surface.

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